



# Class 5: Geometry

# Warm-up Problem 1

A number is increased by 4, multiplied by 5, then decreased by 14. The number is now 41. What was the original number?

We work backwards: Starting from 41, we **add 14** to revert the subtraction.

$$41 + 14 = 55$$

Then, we divide by 5 to revert the multiplication.

$$55 \div 5 = 11$$

Finally, we **subtract 4** to revert the addition.

$$11 - 4 = 7$$

$\therefore$  The original number was 7.

# Warm-up Problem 2

Calculate the following without a calculator or pencil/paper:

(a)  $8 + 98 + 998 + 9998$

(b)  $128 + 186 + 72 - 86$

(c)  $125 \times 32 \times 8$

(d)  $25 \times 125 \times 4 \times 8$

$$\begin{aligned} \text{(a)} \quad 8 + 98 + 998 + 9998 &= (10 - 2) + (100 - 2) + (1000 - 2) + (10000 - 2) \\ &= 10 + 100 + 1000 + 10000 - 8 \\ &= 11102 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 128 + 186 + 72 - 86 &= (128 + 72) + (186 - 86) \\ &= 200 + 100 \\ &= 300 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 125 \times 32 \times 8 &= (125 \times 8) \times 32 \\ &= 1000 \times 32 \\ &= 32000 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 25 \times 125 \times 4 \times 8 &= (25 \times 4) \times (125 \times 8) \\ &= 100 \times 1000 \\ &= 100\,000 \end{aligned}$$

# Homework Take-up – Q1

At a dog training school, dogs can learn to do two tricks: sit and roll over. There are 50 dogs in total and each can do at least one trick. 35 dogs can sit and 25 dogs can roll over. How many dogs can sit and roll over?

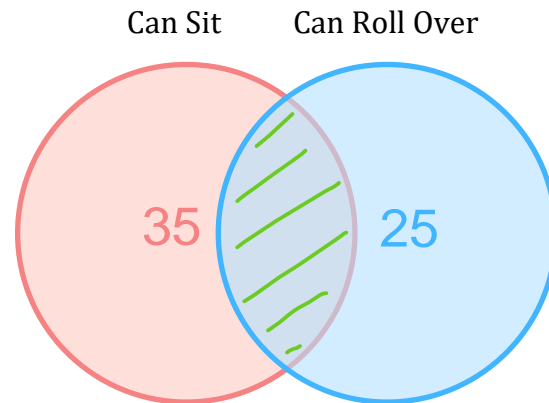
Total number of dogs = (Dogs that can sit) + (Dogs that can roll) – (Dogs that can sit and roll)

$$50 = 35 + 25 - (\text{Dogs that can do both})$$

$$50 = 60 - (\text{Dogs that can do both})$$

$$\text{Dogs that can do both} = 10$$

∴ There are 10 dogs that can sit and roll over.



# Homework Take-up – Q2

There is a bag of marbles with 2 red, 5 blue, and 3 green marbles in each. Someone takes out one marble at random, puts it back in the bag, and draws another marble at random. What is the probability that both colours will be green?

Total number of marbles = 10

Probability of getting green on first draw =  $\frac{3}{10}$

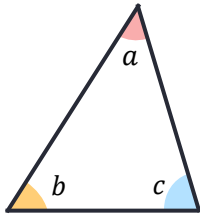
Probability of getting green on second draw =  $\frac{3}{10}$

Probability of getting green on both draws =  $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

∴ There is a  $\frac{9}{100}$  probability that both colours will be green.

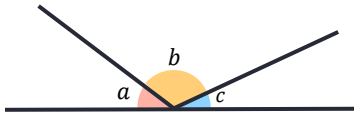
# Angles

Given any triangle, the sum of its 3 interior angles is  $180^\circ$ .



$$\angle a + \angle b + \angle c = 180^\circ$$

**Straight angles:** A straight line always has angle  $180^\circ$  at its intersections.



$$\angle a + \angle b + \angle c = 180^\circ$$

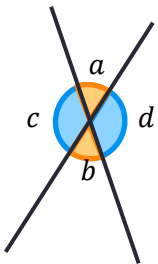
**Right angles:** A right angle is an angle that measures  $90^\circ$ . It is usually denoted by a “square” symbol.



One full rotation around a fixed point measures an angle of  $360^\circ$ .

# Angles

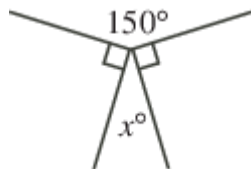
**Diagonal Angles:** Two angles “diagonal” to each other from an intersection of two lines are equal.



$$a = b \text{ and } c = d$$

# Problem

Find the measure of  $x$  in the following diagram.



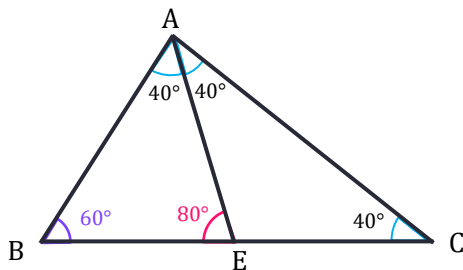
This  $360^\circ$  is made up of one  $150^\circ$  angle, two  $90^\circ$  angles, and  $x$ .

$$\begin{aligned} x &= 360^\circ - 150^\circ - 90^\circ - 90^\circ \\ &= 30^\circ \end{aligned}$$



# Problem

Find the measure of  $\angle AEB$  in the following diagram.



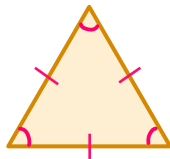
Sum of interior angles of triangle =  $180^\circ$

$$\angle B = 180 - 40 - 40 - 40 = 60^\circ$$

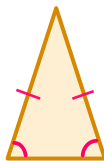
$$\angle AEB = 180 - 40 - 60 = 80^\circ$$

# Triangles

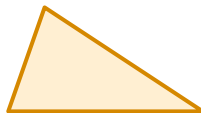
**Equilateral Triangles:** Three equal sides, three equal angles.



**Isosceles Triangles:** Two equal sides, two equal angles.



**Scalene Triangles:** No equal sides, no equal angles.



# Triangles

**Right Triangles:** 1 right angle, 2 acute angles.



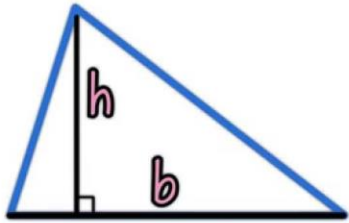
**Acute Triangles:** 3 acute angles



**Obtuse Triangles:** 1 obtuse angle, 2 acute angles



# Area of Triangle

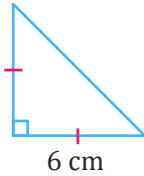


$$\text{Area} = \frac{1}{2} \times b \times h = \frac{bh}{2}$$

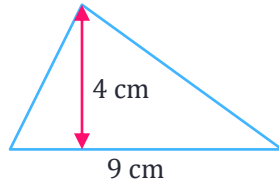
# Problem

Find the area of the triangles:

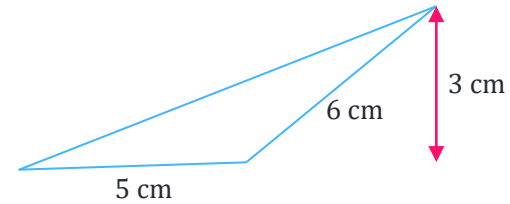
(a)



(b)



(c)



(a) Base = 6 cm, Height = 6 cm

$$\text{Area} = \frac{6 \times 6}{2} = 18 \text{ cm}^2$$

(b) Base = 9 cm, Height = 4 cm

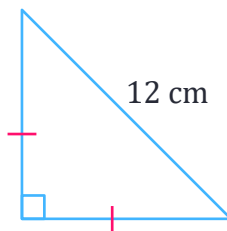
$$\text{Area} = \frac{9 \times 4}{2} = 18 \text{ cm}^2$$

(c) Base = 5 cm, Height = 3 cm

$$\text{Area} = \frac{5 \times 3}{2} = 7.5 \text{ cm}^2$$

# Problem

Find the area of the isosceles right triangle.



## Solution 1:

Area of square = side  $\times$  side

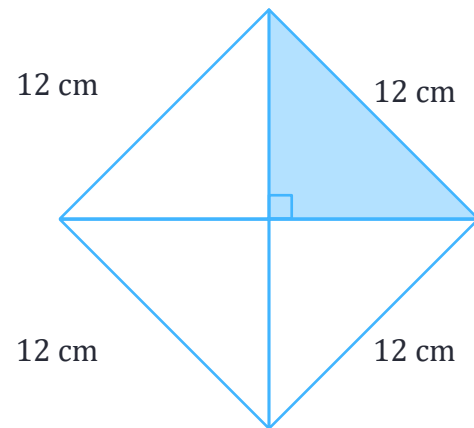
$$= 12 \times 12$$

$$= 144 \text{ cm}^2$$

The triangle is  $\frac{1}{4}$  of the square, so

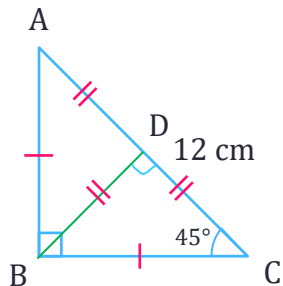
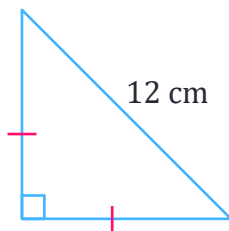
Area of triangle =  $144 \div 4$

$$= 36 \text{ cm}^2$$



# Problem

Find the area of the isosceles right triangle.



## Solution 2:

Since  $\triangle ABC$  is isosceles,  $\angle A = \angle C = 45^\circ$

Since  $\angle BDC = 90^\circ$ ,  $\triangle BDC$  and  $\triangle BDA$  are also isosceles right triangles.

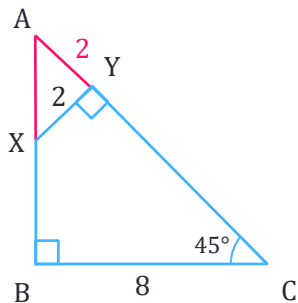
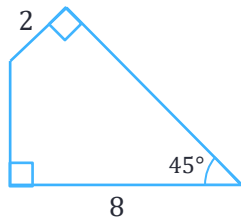
Therefore,  $AD = CD = BD$

Since  $AC = 12$ ,  $AD = CD = \frac{12}{2} = 6$ , so  $BD = 6$ .

$AC$  is the base and  $BD$  is the height, so the area is  $\frac{12 \times 6}{2} = 36 \text{ cm}^2$ .

# Problem

Find the area of the polygon.



Since  $\angle C = 45^\circ$  and  $\angle B = 90^\circ$ ,  $\triangle ABC$  is an isosceles right triangle.

$$\text{Area of } \triangle ABC = \frac{8 \times 8}{2} = 32$$

Since  $\angle AYX = 90^\circ$  and  $\angle A = 45^\circ$ ,  $\triangle AYX$  is an isosceles right triangle as well.

$$\text{Area of } \triangle AYX = \frac{2 \times 2}{2} = 2$$

Area of desired polygon = Area of  $\triangle ABC$  - Area of  $\triangle AYX$

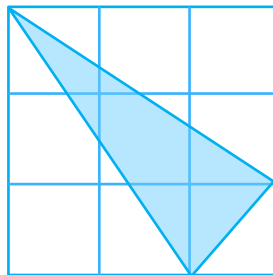
$$= 32 - 2$$

$$= 30$$



# Problem

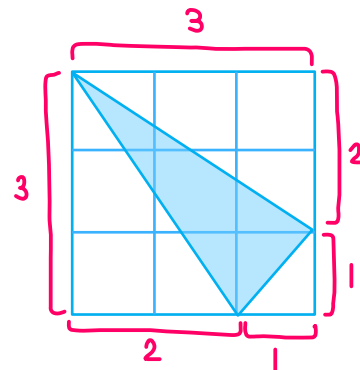
The big square below is made up of 9 small squares of side 1 cm. What is the area of the shaded triangle?



$$\text{Area of big square} = 3 \times 3 = 9 \text{ cm}^2$$

$$\begin{aligned}\text{Total area of unshaded triangles} &= \frac{3 \times 2}{2} + \frac{3 \times 2}{2} + \frac{1 \times 1}{2} \\ &= 3 + 3 + 0.5 \\ &= 6.5 \text{ cm}^2\end{aligned}$$

$$\text{Area of shaded triangle} = 9 - 6.5 = 2.5 \text{ cm}^2$$



# Problem

Four rectangles are drawn inside a larger rectangle. The areas are given inside the diagram. What is the area of rectangle Z?

|   |   |
|---|---|
| W | X |
| Z | Y |

$$\begin{aligned} W &= 6 \text{ cm}^2 \\ X &= 14 \text{ cm}^2 \\ Y &= 35 \text{ cm}^2 \\ Z &= ? \end{aligned}$$

Area of  $W = a \times b = 6$ , area of  $Y = c \times d = 35$

$$a \times b \times c \times d = 6 \times 35 = 210$$

Area of  $X = a \times d = 14$

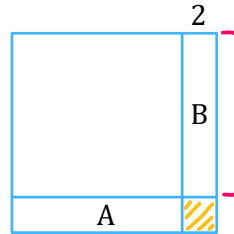
$$\frac{a \times b \times c \times d}{a \times d} = b \times c = \frac{210}{14} = 15$$

Area of  $Z = b \times c = 15$

|       |     |
|-------|-----|
| W $a$ | X   |
| $b$   | $d$ |
| Z $c$ | Y   |

# Problem

The side length of the big square is 2 cm longer than the side length of the small square. The area of the big square is  $40 \text{ cm}^2$  more than the area of the small square. Find the area of the big and small square.



$$\text{Area of shaded square} = 2 \times 2 = 4 \text{ cm}^2$$

We know that (Area of A) + (Area of B) + (Area of shaded square) = 40, so (Area of A) + (Area of B) = 36.

The area of A and B are equal, since they have the same dimensions.

$$\text{Area of A} = \text{Area of B} = \frac{36}{2} = 18$$

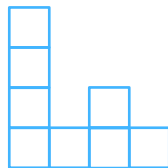
$$\text{Side length of small square} = \frac{18}{2} = 9, \text{ side length of big square} = 9 + 2 = 11$$

$$\text{Area of small square} = 9 \times 9 = \mathbf{81 \text{ cm}^2}$$

$$\text{Area of big square} = 11 \times 11 = \mathbf{121 \text{ cm}^2}$$

# Problem

There are 8 squares, each with side length 2 cm. Find the perimeter of this figure.

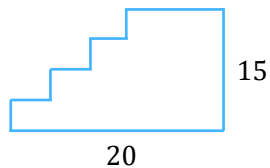


The perimeter is made up of 18 sides of the square.

Each side is 2 cm, so the total perimeter is  $18 \times 2 = 36 \text{ cm}^2$

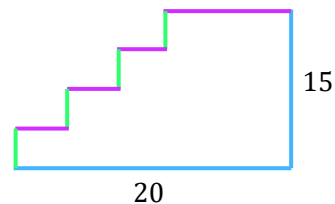
# Problem

There are 8 squares, each with side length 2 cm. Find the perimeter of this figure.



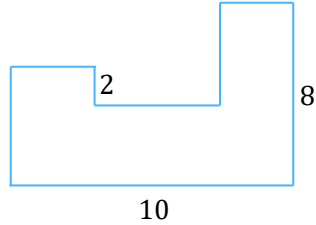
The green lengths add up to 15 and the purple lengths add up to 20.

$$\begin{aligned}\text{Perimeter} &= 15 + 15 + 20 + 20 \\ &= 70\end{aligned}$$



# Problem

There are 8 squares, each with side length 2 cm. Find the perimeter of this figure.



The purple lengths add up to 8, and the green lengths add up to 10. The blue lengths are each 2.

$$\begin{aligned}\text{Perimeter} &= 10 + 10 + 8 + 8 + 2 + 2 \\ &= 40\end{aligned}$$

